Sign of helicity in the α-Ω geodynamo

Xing Wei
Institute of Geophysics,
ETH, Zurich, 8092, Switzerland

November 9, 2010

Abstract

In this paper we discuss the sign of helicity in the α-Ω dynamo and propose a wave-Ω model in which the α effect in the geodynamo is induced by helical wave but not helical flow as in the solar dynamo. We then tentatively interpret the mechanisms of the Earth's magnetic westward drift, tilt angle, and dipole reversals.

1 Introduction

The Earth's magnetic field is generated and maintained by the motion of electrically conducting liquid iron in the outer core, that is the self-sustained geodynamo action. The thermal and compositional buoyancy forces drive convective motions in the fluid outer core while the Earth's rotation promotes the fluid helical motions, and these convective and helical motions shear and twist the magnetic field lines, intensifying the magnetic field and offsetting the effects of diffusion.

The law of angular momentum conservation indicates that the convection in the Earth's fluid core tends to drive an eastward differential rotation and consequently the interaction between the convection and magnetic field makes the inner core rotate slightly faster than the mantle, which has been verified by both seismic data [27] and computer simulations [7, 8] (though the super-rotation of the inner core is still controversial [23]). This large-scale differential rotation shears the dipolar field lines such that a strong azimuthal field is created, which is called the Ω effect. In the meanwhile, the turbulent eddies in the rotating fluid have helical motions arising from the Coriolis force as they rise and sink, and these eddies then twist the azimuthal field lines created by the Ω effect to induce the dipolar field lines, which is called the α effect [17]. A loop between dipolar and azimuthal fields is therefore established, and this is the classic α-Ω geodynamo model [10, 18].

Another possibility is the α^2 geodynamo model in which both the two processes in the loop are implemented by the α effect. Most simulations support the α^2 model [16], but these simulations work in the regime of high Ekman number (a dimensionless number to measure the ratio of viscous and rotational effects) which is far from the regime of the Earth's core, and in the

simulations run in the regime of low Ekman number the striking differential rotation emerges due to the thermal wind and the Stewartson shear layer [19].

Unlike the Sun, the turbulence in the Earth's core is highly anisotropic due to the Coriolis and Lorentz forces [4], and the large magnetic diffusivity of liquid iron can suppress the turbulent induction effect [18]. These imply that the small-scale turbulent α effect may not be significant to the geodynamo.

2 Sign of helicity

In the mean field theory of the α-Ω dynamo [11], the electromotive force (e.m.f) induced by the small-scale turbulent α effect can be modeled as

$$\langle \mathbf{u} \times \mathbf{b} \rangle_\phi = \alpha \langle \mathbf{B} \rangle_\phi, \tag{1}$$

where $\mathbf{u}$ and $\mathbf{b}$ are the turbulent velocity and field respectively, the symbol $\langle\rangle$ represents the resemble average and the subscript ϕ represents the azimuthal component. We now use Eq.(1) to investigate the sign of helicity $h = \mathbf{u} \cdot (\nabla \times \mathbf{u})$.

The azimuthal field induced by the Ω effect arising from an axisymmetric large-scale shear flow is also axisymmetric and in a large scale. In the Ω effect induced by the eastward differential rotation, if the original dipolar field points to the north then the azimuthal field points to the west in the northern hemisphere (denoted by N in the following text) and to the east in the southern hemisphere (denoted by S in the following text). On the contrary, if the original dipolar field points to the south then the signs of azimuthal field reverse. We take the original dipolar field pointing to the north for example. $\langle \mathbf{B} \rangle_\phi$ is negative in N and positive in S, therefore, if α is positive in N and negative in S then the e.m.f. $\langle \mathbf{u} \times \mathbf{b} \rangle_\phi$ will be negative in both hemispheres to induce a new dipolar flied pointing to the south which can weaken the original dipolar field, and consequently the dynamo will be oscillatory. However, if α is negative in N and positive in S then the original dipolar field will be strengthened by the induced dipolar field pointing to the north and the dynamo will be non-oscillatory. In the other case in which the original dipolar field points to the south, the result is the same, i.e. α which is + in N and - in S generates an oscillatory dynamo and α which is - in N and + in S generates a non-oscillatory dynamo. On the other hand, α is associated with the fluid helicity $h = \mathbf{u} \cdot (\nabla \times \mathbf{u})$ [12, 3]. Both of them are pseudo-scalars but their signs are opposite to each other. Accordingly, if h is - in N and + in S then the dynamo is oscillatory whereas if h is + in N and - in S then the dynamo is non-oscillatory.

In the spherical rotating flow, when a fluid particle is driven by the convection to move radially, the Coriolis force will bend it to cause a helical flow. It is readily confirmed that the helicity of this helical flow is - in N and + in S, thus the helical flow should create an oscillatory dynamo, which is just the situation of the solar dynamo. However, the geodynamo is non-oscillatory (though it has dipolar reversals in quite a large time scale), which implies that the α effect in the geodynamo is not induced by the helical flow. An alternative is helical wave.

It is well known that helical waves can induce an α effect [13, 14, 24, 28]. Suppose that we have a helical wave $\mathbf{u}$ of amplitude u_0, wave vector $\mathbf{k}$ and

frequency ϖ. In a local Cartesian coordinate system (X, Y, Z) where the X axis is chosen to be parallel to $\mathbf{k}$, the definition of helical wave $\nabla \times \mathbf{u} = \pm k\mathbf{u}$ gives $\mathbf{u} = (0, u_0 \cos(kx - \varpi t), \mp u_0 \sin(kx - \varpi t))$, where the top sign represents a right-handed helical wave (positive h) and the bottom sign left-handed (negative h). Substituting this expression into the perturbed induction equation

$$\frac{\partial \mathbf{b}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}_0) + \eta \nabla^2 \mathbf{b}, \tag{2}$$

where $\mathbf{B}_0$ is the background field (the azimuthal field caused by the Ω effect) and taken to be locally uniform, $\mathbf{b}$ is the field induced by the passage of the helical wave and η is the magnetic diffusivity, we have the induced e.m.f. [14]

$$\mathbf{u} \times \mathbf{b} = \mp \frac{\eta k u_0^2 \mathbf{B}_0 \cdot \mathbf{k}}{\varpi^2 + \eta^2 k^4} \mathbf{k}, \tag{3}$$

where the top and bottom signs are associated with the right-handed and left-handed helical waves respectively. Equation (3) shows that a helical wave can create an electric current loop (the local X axis can be thought to form a loop) which will in turn induce a dipolar field. To maintain a non-oscillatory dynamo, $\mathbf{B}_0 \cdot (\mathbf{u} \times \mathbf{b})$ should be - in N and + in S, such that right-handed helical waves propagate in N and left-handed in S, namely the helicity of helical wave is + in N and - in S, which is opposite to the sign of helicity in helical flow. Now we have confirmed that the sign of helicity of helical wave is consistent with the above analysis based on the mean field theory.

3 Illustrations of some geomagnetic phenomena

In the Earth's fluid core there exist various hydrodynamic and hydromagnetic waves, inertial wave, Rossby wave, magnetostrophic wave, magneto-Rossby wave, etc., which has been verified by laboratory experiments [22, 15]. Based on these helical waves, a wave-Ω geodynamo model has already been proposed by Davidson [5], in which the azimuthal field is generated by differential rotation acting on the dipolar field and the dipolar field is regenerated from the azimuthal field by helical waves propagating through the interior of the Earth's fluid core. In addition, this wave-Ω dynamo has also been found in the numerical simulations [20, 26], in which Rossby wave induces the α effect.

A natural question to ask is where these waves come from. Convection itself can generate some instabilities and the propagation of these convective instabilities is wave motions. Another possibility is topography waves. When the convective flows move over the bumps at the fluid core boundary, either the core-mantle boundary or the inner core boundary or both, some topography waves will be triggered. Once these waves are created they will develop into helical waves due to the Earth's rotation and contribute to the α effect in dynamo action.

We now focus on two particular helical waves, the inertial wave arising from the Earth's rapid rotation and the magnetostrophic wave arising from the combined effect of rapid rotation and magnetic field [2]. Inertial wave is fast and short whereas magnetostrophic wave is slow and long. The phase speed

of magnetostrophic wave is comparable to that of the geomagnetic westward drift, and this implies that the westward drift can be caused by the phase velocity of magnetostrophic waves [9]. Though it is possible that the westward drift can be caused by a westward flow at the fluid core surface [1], the most recent observations support the wave idea because the magnetic fluxes drifting westward in the observations are more likely to be associated with wave motion rather than convective flow [6]. And this idea of wave driven westward drift has been accomplished by numerical simulations [21]. One may have noticed that according to Eq.(3) $\mathbf{k}$ can be either eastward or westward but has no preference for westward. This can be explained by the β plane effect which drives the magnetostrophic waves westward and the inertial waves eastward due to the geometry of thick spherical shell [9]. Then another question arises, why cannot we detect the influence of phase velocity of inertial wave on the geomagnetic filed, namely the eastward drift? This can be explained by the skin effect of the mantle. Because the inertial wave oscillates too fast and the conductivity of the mantle is quite small, the influence of inertial wave on the magnetic field cannot deeply penetrate into the mantle but is screened by the mantle. On the contrary, because the magnetostrophic wave oscillates very slowly the westward drift can be observed on the Earth's surface.

Moreover, the group velocity of inertial wave is perpendicular to its phase velocity and thus perpendicular to the e.m.f. $\mathbf{u} \times \mathbf{b}$ (Eq.3), which implies that the inertial wave propagates along the axis of the induced dipolar field. Therefore, the tilt angle between the axis of the Earth's rotation and the axis of the geomagnetic dipolar field is associated with the propagation of inertial wave.

To end this section we tentatively illustrate the mechanism that triggers the geomagnetic dipole reversals. Some helical waves with "inappropriate" wavelengths and periods propagate too fast and are reflected by the boundary. The sign of helicity of these reflected helical waves is opposite to that of incident waves such that these waves induce an opposite dynamo action, and consequently the dipolar field will be weakened, which is similar to the situation in an oscillatory dynamo. Because the surface of the boundary is spherical but not flat, the reflected waves will not go back along the track of the incident waves, which can destroy the dipolar structure of the geomagnetism. This illustration is in agreement with the observations that the geomagnetism becomes non-dipolar during reversals. After these reflected waves decay away, a new non-oscillatory dynamo is established to work but the dipolar field has already reversed.

4 Conclusions and further work

We discuss the sign of helicity h in the α-Ω dynamo. If h is - in N and + in S then the dynamo is oscillatory whereas if h is + in N and - in S then the dynamo is non-oscillatory. The former happens in the solar dynamo in which the α effect is induced by turbulent helical flow, and the latter happens in the geodynamo in which the α effect is induced by helical wave. We also verify that the sign of helicity of helical wave is consistent with the sign of helicity in the non-oscillatory α-Ω dynamo. We then employ the helical wave α effect to illustrate the geomagnetic westward drift, tilt angle and dipole reversals.

Some further work is anticipated. We can experimentally implement this helical wave α effect by designing a spherical dynamo setup in which we observe how the helical waves propagate, reflect, and in the meanwhile, induce the magnetic field. We can also numerically implement by solving the fully three-dimensional and strongly non-linear magnetohydrodynamic (MHD) equations.

Acknowledgements

This work is based on my PhD thesis [25] and I thank my supervisor Peter Davidson for his original wave-Ω model and great helps on my study. I also thank Rainer Hollerbach, Andrew Jackson and Chris Finlay for their helpful discussions on the geodynamo theory, the skin effect and the β plane effect, respectively.

References

[1] E. C. Bullard, C. Freeman, H. Gellman, and J. Nixon. The westward drift of the earth's magnetic field. *Phil. Trans. R. Soc. A*, 243:61–92, 1950.

[2] S. Chandrasekhar. *Hydrodynamic and Hydromagnetic Stability*, pages 197–198. Oxford Clarendon Press (Oxford UK), 1961.

[3] P. A. Davidson. *An introduction to MHD*, pages 181–183. Cambridge University Press (Cambridge UK), 2001.

[4] P. A. Davidson. *Turbulence: An Introduction for Scientists and Engineers*, pages 548–551. Oxford University Press (Oxford UK), 2004.

[5] P. A. Davidson. *Turbulence: An Introduction for Scientists and Engineers*, pages 561–562. Oxford University Press (Oxford UK), 2004.

[6] C. C. Finlay and A. Jackson. Equatorially dominated magnetic field change at the surface of earth's core. *Science*, 300:2084–2086, 2003.

[7] G. A. Glatzmaier and P. H. Roberts. A three-dimensional convective dynamo solution with rotating and finitely conducting inner core and mantle. *Phys. Earth Planet. Inter.*, 91:63–75, 1995.

[8] G. A. Glatzmaier and P. H. Roberts. A three-dimensional self-consistent computer simulation of a geomagnetic field reversal. *Nature*, 377:203–209, 1995.

[9] R. Hide. Free hydromagnetic oscillations of the earth's core and the theory of the geomagnetic secular variation. *Phil. Trans. R. Soc. A*, 259:615–647, 1966.

[10] R. Hollerbach. On the theory of the geodynamo. *Phys. Earth Planet. Inter.*, 98:163–185, 1996.

[11] F. Krause and K. H. Radler. *Mean-field magnetohydrodynamics and dynamo theory*, page 22. Akademie-Verlag (Berlin Germany), 1980.

[12] F. Krause and K. H. Radler. *Mean-field magnetohydrodynamics and dynamo theory*, pages 25–31. Akademie-Verlag (Berlin Germany), 1980.

[13] H. K. Moffatt. Dynamo action associated with random inertial waves in a rotating conducing fluid. *J. Fluid Mech.*, 44:705–719, 1970.

[14] H. K. Moffatt. *Magnetic Field Generation in Electrically Conducting Fluids*, pages 162–165. Cambridge University Press (Cambridge UK), 1978.

[15] M. D. Nornberg, H. Ji, E. Schartman, A. Roach, and J. Goodman. Observation of magnetocoriolis waves in a liquid metal taylor-couette experiment. *Phys. Rev. Lett.*, 104(074501), 2010.

[16] P. Olson, U. Chritensen, and G. A. Glatzmaier. Numerical modeling of the geodynamo: Mechanisms of field generation and equilibration. *J. Geophys. Res.*, 104(B5):10383–10404, 1999.

[17] E. N. Parker. Hydromagnetic dynamo models. *Astrophys. J.*, 122:293–314, 1955.

[18] P. H. Roberts and G. A. Glatzmaier. Geodynamo theory and simulations. *Rev. Mod. Phys.*, 72:1081–1123, 2000.

[19] A. Sakuraba and P. H. Roberts. Generation of a strong magnetic field using uniform heat flux at the surface of the core. *Nature Geoscience*, 2:802–805, 2009.

[20] N. Schaeffer and P. Cardin. Quasi-geostrophic kinematic dynamos at low magnetic prandtl number. *Earth Planet. Sci. Lett.*, 245:595–604, 2006.

[21] D. Schmitt. A new mechanism to understand the westward drift of the Earth's magnetic field (in preparation). 2010.

[22] D. Schmitt, T. Alboussiere, D. Brito, P. Cardin, N. Gagniere, D. Jault, and H. C. Nataf. Rotating spherical couette flow in a dipolar magnetic field: experimental study of magneto-inertial waves. *J. Fluid Mech.*, 604:175–197, 2008.

[23] A. Souriau and G Poupinet. Inner core rotation: a critical approach. In V. Dehant, K. C. Creager, S. Karato, and S Zatman, editors, *Earth's Core: Dynamics, Structure, Rotation (Geodynamics Series 31)*, pages 65–82. American Geophysical Union (Washington US), 2003.

[24] A. Tilgner. Dynamo action with wave motion. *Phys. Rev. Lett.*, 100(128501), 2008.

[25] X. Wei. *Rotating MHD in a spherical shell.* PhD thesis, University of Cambridge, 2009.

[26] X. Wei and R. Hollerbach. Kinematic dynamo induced by Rossby waves in a spherical Couette flow (in preparation). 2010.

[27] J. Zhang, X. Song, Y. Li, P. G. Richards, X. Sun, and F. Waldhauser. Inner core differential motion confirmed by earthquake waveform doublets. *Science*, 309:1357–1360, 2005.

[28] R. A. Zuniga, F. Plunian, and K. H. Radler. Rossby waves and α effect. *Geophys. Astrophys. Fluid Dyn.*, 103:375–396, 2009.